HOT-FILM ANEMOMETRY IN AQUEOUS NaCl SOLUTIONS

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Abstract – An experimental investigation was performed on the calibration of cylindrical hot-film sensors in aqueous NaCl solutions. The results are consistent with Kramers' law provided the Reynolds number exponent is reduced to 0.36 and correction is made for conduction to the quartz fiber backing. A modified King's law appears to provide a useful approximation to the usual Kramers' law for practical applications of hot-film sensors in flows of variable density. The modified King's law provides a simple method for estimating the magnitude of errors introduced when variable density effects are ignored in the velocity calibration of a hot-film sensor in saline solutions.

NOMENCLATURE

A,	$\pi L R k_w (0.42\,Pr^{0.2} + 0.284)$;
B,	$0.57\,\pi L R k_w Pr^{0.33}\,(\rho_w^2 d/\mu_w)^n$;
c,	thermal heat capacity;
d,	diameter of film sensor;
E,	voltage across hot-film sensor;
k,	thermal conductivity;
K,	dimensionless conductivity factor;
L,	length of film sensor;
Nu,	Nusselt number, $E^2/(\pi L R k_w \Delta T)$;
Pr,	Prandtl number, $c_w \mu_w / k_w$;
R,	heated resistance of film sensor;
Re,	Reynolds number, $\rho_w U d/\mu_w$;
T,	thermodynamic temperature;
U,	velocity;
Y,	Kramers' parameter.

Greek symbols

ρ,	density;
μ,	dynamic viscosity;
ΔT,	temperature difference, $T_f - T_a$;
Δu,	overall velocity difference;
$\Delta \rho$,	overall density difference.

Subscripts

a,	ambient conditions;
b,	quartz backing of film sensor;
c,	corrected;
f,	hot-film sensor;
n,	velocity and Re exponent;
o,	observed;
r,	reference condition;
w,	saline water properties evaluated at film temperature, $(T_f + T_a)/2$;

291,	parameter evaluated at $T = 291°K$;
1000,	parameter evaluated at $\rho_w = 1000\ \text{kg m}^{-3}$.

1. INTRODUCTION

HOT-FILM anemometry is an important tool for the measurement of velocity fields in liquids. While liquid and gas hot-film anemometry are similar, there are additional difficulties encountered in film sensor operation in liquids. Sensor contamination can be a serious problem umless the liquid is filtered well; formation of gas bubbles on the sensor requires careful deaeration of dissolved gases. In addition, the hot-film sensor must be operated at temperatures well below the boiling point resulting in a combined sensitivity to velocity and temperature. Hot-film anemometry in homogeneous liquids has been discussed in considerable detail in the literature [1–4]. A strong interest in atmospheric and oceanographic fluid mechanics has led to the wide use of hot-film anemometry in experimental studies of stratified flows [5, 6]. Common salt, NaCl, dissolved in water can produce up to a 20% variation in liquid density. Aqueous saline solutions provide an inexpensive, convenient, and safe method for stratifying laboratory flows. Despite a rise in popularity of laser anemometry, hot-film sensors are still the most common form of velocity transducer used in measurements of stratified flows. The sensors are commercially available with an extra thick coating of quartz which insulates the metallic sensor from the highly conducting salt water. While a 20% density variation is possible in saline solutions, typically the variations are at most a few percent, and it is usually assumed that hot-film velocity measurements are not seriously contaminated by density variations. Private communications with several investigators indicate that informal testing of this assumption has been performed, but no discussion of mixed velocity and density sensitivities of hot-film anemometers operated in liquids appears in the literature. In a recent

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review article on stratified flows, Maxworthy and
Browand [7] caution that hot-films do depend on
local fluid properties and independent measurements
may need to be made of such quantities as temperature
and density in order to properly interpret the anem-
ometer output. Research in progress in the University
of California, San Diego continuous-return stratified
flow water tunnel currently employs density variations
up to 5% and larger density variations may be used in
the future. Under these circumstances it is important
that some consideration be given to a possible mixed
sensitivity of the hot-film anemometer signal to vel-
ocity and density.

Experiments were conducted in a calibration facility
to determine the steady-state heat-transfer characteris-
tics of cylindrical hot-film sensors operated in aqueous
saline solutions with density variations of 17%, very
near the maximum density variation possible. The
form of the dimensionless heat-transfer correlation
was examined to determine whether modifications
would be required in saline solutions. Once confidence
was established in a heat-transfer correlation for
cylindrical hot-film sensors, estimates were made in
order to determine whether velocity signals were
contaminated by a sensitivity to density variations. In
addition, consideration was given to hot-film sensi-
tivity to ambient temperature variations.

2. EXPERIMENTAL ARRANGEMENT

A sketch of the experimental facility is shown in Fig.
1. The test loop was filled with a saline solution of the
chosen density from a set of storage tanks. The hot-film
sensors were supported at the exit of a 0.0254 m I.D.
pipe. The water in the test loop was circulated with a
positive displacement pump through a sharp-edged
orifice into the test tank. The fluid velocity was
measured with a differential manometer connected
across an orifice plate. The orifice plate–manometer
system was calibrated by first reading the differential
pressure and then diverting the water solution through
the overflow valve and timing the flow into a measured
volume. Use of the positive displacement pump in-
sured that the flow rate would not change significantly
during the calibration process. The manometer calib-

rations were run using the range of salt water solutions
used in the hot-film experiments. Since the manometer
working fluid was always the same density as the fluid
in the test loop, the manometer calibrations were
found to be independent of density as expected. The
hot-film sensors were Thermo-Systems, Inc., 1210-20
NACL and were operated at an overheat ratio of 1.05
by a DISA 55M01 constant-temperature anemometer
system. The bridge output voltage of the anemometer
system was read on a Hewlett–Packard 5326B
counter-timer averaging voltmeter. Fluid densities
were measured with a set of Brooklyn Thermometer,
Inc. ASTM hydrometers accurate to ± 0.0005 s.g.

Initial runs were made in the test loop without
thermal control of the fluid temperature. Sub-
sequently, a cooling system was installed in the test
loop to counter the temperature rise caused by the
pump. The hot-film sensors were mounted so that the
sensing portion of the probe was recessed appro-
ximately 0.02 m into the pipe in order to minimize the
effects of recirculation in the test tank at low speeds.
Despite these precautions, some difficulty was en-
countered at Reynolds numbers less than 5, and these
data points were not included in the analysis.

3. DATA ANALYSIS AND RESULTS

The first step in the analysis of the experimental data
was a comparison between the observed Nusselt
number with that predicted from the empirical cor-
relation suggested by Kramers [8]

$$Nu = 0.42\,Pr^{0.2} + 0.57\,Pr^{0.33}\,Re^{0.45} \qquad (3.1)$$

A linear regression analysis of the hot-film measure-
ments gives

$$Nu_o = 0.66\,Nu + \text{const.} \qquad (3.2)$$

with a correlation coefficient of 99% and a relative
standard deviation on the slope of 1.3%. The differ-
ences between the observed and empirical Nusselt
numbers detected by equation (3.2) suggest that some
modifications to the Kramers' parameters are required
to properly represent the correlation of equation (3.1)
with the measurements of cylindrical films. Heat loss
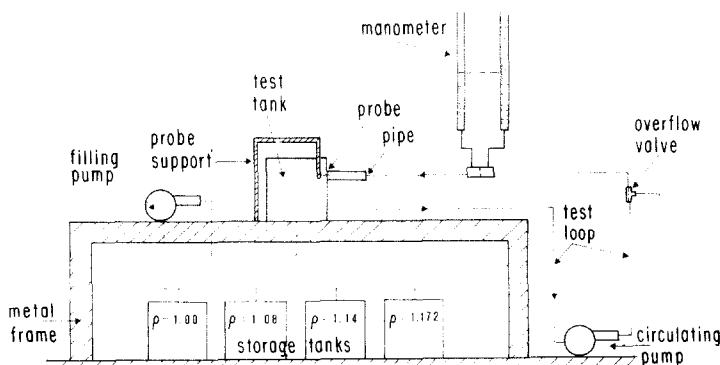from the film to the quartz backing material could
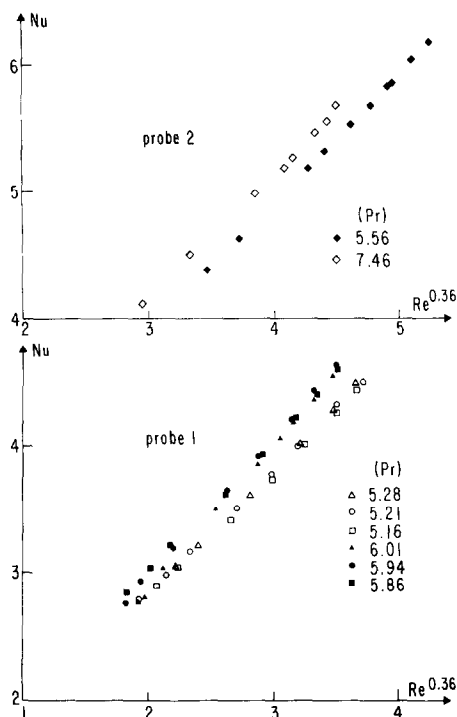

Fig. 1. UCSD variable density calibration facility.

Fig. 2. Measured Nu vs $Re^{0.36}$ for various Pr numbers: (a) probe 1, (b) probe 2.

account for the constant offset in equation (3.2). Some authors [9, 10] have related this constant to a dimensionless heat-conduction term $K(k_b/k_w)$. The slope 0.66 in equation (3.2) does imply that a change must be made in the coefficients or exponents of the Kramers' law. Several authors [4, 11] have reported that for cylindrical hot-film sensors operated in liquids, the observed Nusselt numbers lie somewhat below the empirical Kramers' law. Since we are ultimately interested in using the empirical heat-transfer correlation for mixed velocity and density sensitivity estimates over a range of salt concentrations, the Kramers' parameters should be adjusted for a better fit to our measurements before sensitivity estimates are made.

We have investigated the Reynolds number exponent by starting with a generalized Kramers' law. When the Prandtl number is constant, equation (2.1) can be simplified to

$$Nu = a + bRe^n \qquad (3.3)$$

where a is the Nusselt number in quiescent fluid. Following the literature we let $a = zNu_o$, where Nu_o is the observed Nusselt number for no flow, and z is a parameter which is adjusted during the exponent optimization. Typically z lies between 0 and 1. Rewriting equation (3.3) in logarithmic form using this notation we obtain

$$\log (Nu/zNu_o - 1) = n \log Re + \text{const} \qquad (3.4)$$

which is an equation with a slope equal to the exponent of the Reynolds number. The optimization for n is

performed by adjusting the parameter z maximizing the correlation coefficient of the relative regression line.

The data were arranged in eight sequences, two of which have a strictly constant Prandtl number, and the Prandtl number was nearly constant for the other six. The various Prandtl numbers vary both because of temperature and density differences for each experimental run. A regression analysis was then performed using equation (3.4) for each of the Prandtl number sequences giving an average slope or Reynolds number exponent of 0.36. No distinct trend in the exponent with Prandtl number was observed. The measured Nusselt numbers plotted against $Re^{0.36}$ are shown in Figs 2(a) and 2(b). Each data set for a given Prandtl number appears to be well represented by a straight line. A linear regression was performed on each set of Prandtl number data to find the coefficients a and b appearing in equation (3.3). The set of coefficients are really an empirical function of Prandtl number. The variation of the coefficients a and b were compared with the corresponding terms in Kramers' law $(0.42\,Pr^{0.2}$ and $0.57\,Pr^{0.33}$, respectively). The slopes are in excellent agreement, but there appears to be a nearly constant difference for the intercept, a, for each Prandtl number sequence. This extra Nusselt number, equal to 0.284 on average, is probably explained by the heat losses from the heated film sensor to its quartz backing. Using the dimensionless conduction formulation, $K(k_b/k_w)$, we obtain $K = 0.408$ for our cylindrical hot-film sensors. Ling [9] obtained $K = 2.8$ for wedge-shaped sensors and Bonis and van Thinh [10] found $K = 1.55$ for conical sensors.

The data analysis discussed above suggests that a modified Kramers' law

$$Nu_c = 0.42\,Pr^{0.2} + 0.57\,Pr^{0.33}\,Re^{0.36} \qquad (3.5)$$

with the corrected Nusselt number given by

$$Nu_c = Nu_o - 0.284 \qquad (3.6)$$

should give a good representation of the heat-transfer characteristics of cylindrical hot-film sensors in both water and salt water solutions. This analysis represents a form of nonlinear regression. Formally a multiple regression of the observed Nusselt number could have been using two predictors $X_1 = Pr^{0.2}$ and $X_2 = Pr^{0.33}\,Re^{0.36}$, but the relatively scarce set of Prandtl numbers would probably prevent achieving significantly better results. We chose to test the validity of our regression procedure by introducing the Kramers' parameter

$$Y_c = (Nu_c - 0.42\,Pr^{0.2})/Pr^{0.33} \qquad (3.7)$$

where once again we have used the Nusselt number corrected for heat conduction losses to the quartz backing. If equation (3.5) accurately correlates the data, a plot of Y_c vs $Re^{0.36}$ should collapse the data onto a single line passing through the origin with a slope of 0.57. The data are presented in this manner in Fig. 3. The agreement between the data and the
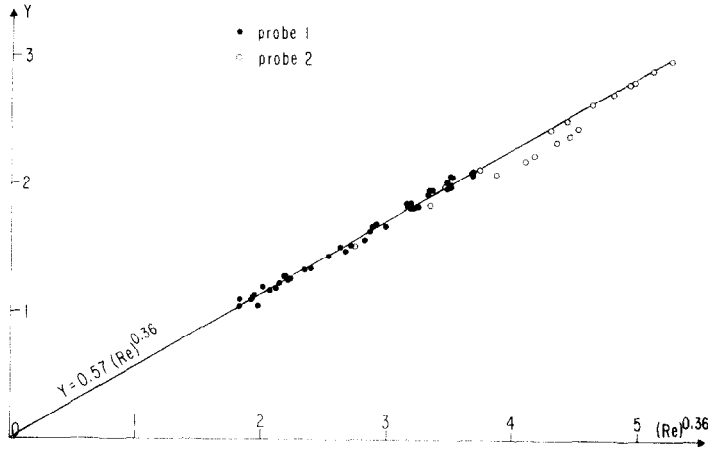
FIG. 3. Measured Kramers' parameter Y vs $Re^{0.36}$ for probes 1 and 2.

equation

$$Y_c = 0.57 \, Re^{0.36} \qquad (3.8)$$

are good except for the density $1172 \, \text{kg m}^{-3}$. The deviation of the points at this density is almost certainly caused by an observed drift of the hot-film sensor as we were unable to repeat the measurements made with this sensor at lower salt concentrations. A least-square regression to the data of Fig. 3 gives a slope of 0.555 with a correlation of 99.6%, and if the $\rho_w = 1172 \, \text{kg m}^{-3}$ data are rejected, then the resulting slope is 0.571. It appears that the present data are well represented by the modified Kramers' law given by equation (3.5).

4. PRACTICAL CONSIDERATIONS

This study has shown that the slightly modified Kramers' law given by equation (3.5) collapses the hot-film anemometer data over useful ranges of velocity,

density, and temperature. Unfortunately, equation (3.5) is rather complicated and does not lend itself to easy application in most experiments. To this end, we have explored the possibility of using a greatly simplified version of equation (3.5) which still retains the important parametric behavior of the density contribution. The simplified equation is given by

$$\frac{E^2}{\Delta T} = A + B \left(\frac{U}{\rho_w} \right)^n \qquad (4.1)$$

where the parameters A and B are given from equation (3.5) by

$$A = \pi L R k_w (0.42 \, Pr^{0.2} + 0.284) \qquad (4.2)$$

$$B = 0.57 \, \pi L R k_w Pr^{0.33} \left(\frac{\rho_w^2 d}{\mu_w} \right)^n \qquad (4.2a)$$

and $n = 0.36$ for this particular choice of hot-film sensor. Both A and B are at most weakly dependent on
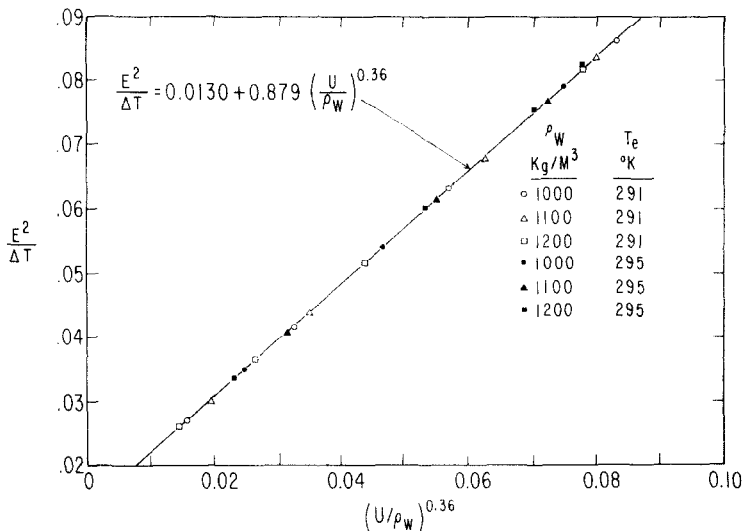

FIG. 4. Comparison of values computed from modified Kramers' law [equation (3.5)] with a least-square fit using a modified King's law representation [equation (4.1)].

temperature and density. The representation of equation (4.1) is suggested by a systematic numerical evaluation of the Kramers' law in equation (3.5). The properties were varied in the ranges $291 < T_a < 295$ K and $1000 < \rho_w < 1200$ kg m^{-3}, and the voltage E was calculated from equation (3.5) over a velocity range of $0.01 < U < 1$ m s^{-1}. The results are shown in Fig. 4. Note that the symbols represent particular choices of the parameters used to evaluate equation (3.5) and do not represent laboratory data. Other physical parameters such as hot-film geometry and properties and operating conditions were chosen to match the experimental conditions described in Section 2. The line in Fig. 4 is the linear regression of equation (4.1) to the evaluated points. Observe that the numerical data points cluster tightly around the line representing equation (4.1) for the entire range of density, temperature and velocity. Similar results are obtained when the overheat is permitted to vary; the magnitudes of A and B increase by 6 and 12%, respectively, when the overheat is increased from 0.2 to 0.10, but equation (4.1) remains a valid function form over the whole range of overheats. It appears justified to conclude that equation (4.1) can provide a useful approximation to the full Kramers' law in saline solutions with moderate temperature variations.

Experience has shown that the exponent n is highly probe dependent with values in the range $0.2 < n < 0.4$. It is important to consider the variation of the parameters A and B of equation (4.2) with variations in temperature and density for this range of exponents. This is done by examining curves of A and B nondimensionalized by A and B evaluated at the fresh water density. Note that only B is dependent on n. The resulting density variations of A and B are shown in Fig. 5. We see that for an exponent of $n = 0.2$, the variation in B/B_{1000} becomes large for $\rho_w > 1100$ kg m^{-3}. Note that while the variation in A/A_{1000} appears large, the absolute value of the parameter A is small and B dominates over most of the velocity range. A similar study of A and B for variations of ambient

temperature T_a of 4 K result in changes of 0.3% in A/A_{291} and 0.5% in B/B_{291}. The variations in A and B due to temperature are nearly an order of magnitude smaller than those due to density and are insensitive to changes in overheat from 0.2–0.10. The simplifications introduced in equation (4.1) appear to work well at large exponents, and the quality of the approximation is degraded with small exponents, especially at densities greater than 1100 kg m^{-3}. Particular applications of equation (4.1) may require an investigation of the behaviors of A, B and n for experiments with conditions which are markedly different from those reported in this study.

It should be noted that even though A and B are only weakly dependent on temperature, accurate measurement of velocity requires an instantaneous measurement of the ambient temperature local to the hot-film. The temperature measurement is difficult if the ambient temperature change is more than just a slow drift (time scales > 10 s).

Finally we note that it is easy to show that if the assumptions required for equation (4.1) are valid, then the error in measurements of the mean velocity are affected by density according to

$$\frac{\Delta u}{U} = \frac{\Delta \rho}{\rho}, \qquad (4.3)$$

so if a hot-film sensor were traversed through a stratified flow field with an overall density variation of 5%, then the mean velocity measurement would be in error by about the same amount, provided no corrections were made for the effects of density. The results of this study are consistent with the private communications mentioned previously provided the overall density variations are small. Rarely do laboratory produced stratified flow experiments use maximum density variations exceeding a few percent, and thus density effects on measurements of U can usually be considered negligible. Equation (4.1) provides a useful simplification of the full Kramers' law (equation (3.5)) for practical hot-film calibrations in aqueous
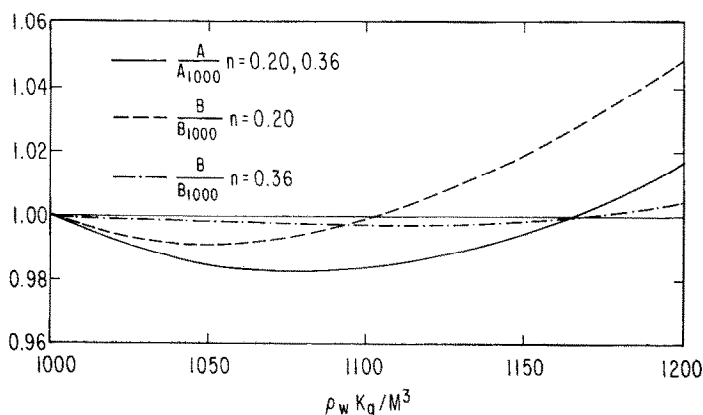

FIG. 5. Variation of King's law parameters A and B in equation (4.1) with density and Reynolds number exponent.

NaCl solutions where the property variations require correction for density or temperature changes.

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APPENDIX: PROPERTIES OF AQUEOUS NaCl SOLUTIONS

Data for c_w, k_w and μ_w as functions of saline concentration (at 293.16 K) were taken from Kaufmann [11]. Quadratic regressions were used to represent the properties as functions of density in SI units giving

$$c_w = 2.3839 \times 10^4 - 32.40\,\rho_w + 1.2738 \times 10^{-2}\,\rho_w^2 \qquad (A1)$$

$$k_w = 0.95776 - 5.4753 \times 10^{-4}\,\rho_w + 1.8850 \times 10^{-7}\,\rho_w^2 \quad (A2)$$

$$\mu_w = 2.331 \times 10^{-2} - 4.454 \times 10^{-5}\,\rho_w + 2.225$$
$$\times 10^{-8}\,\rho_w^2 \quad (A3)$$

with correlation coefficients greater than 99%. The variations of the properties with temperature were approximated by using formulae valid for fresh water and extending them to the saline case in the following manner: the fresh water variation of c_w with temperature can be neglected without introducing significant error. The fresh water variation of k_w with temperature is given by

$$k_w(T) = k_w(293.16) + b(T - 293.16) \qquad (A4)$$

where $k_w(293.16 \text{ K}) = 0.559 \text{ W m}^{-1}\text{K}^{-1}$ and $b = 1.85 \times 10^{-3}\text{ W m}^{-1}\text{K}^{-2}$ for $270 \leq T \leq 320$ K. We extend the fresh water temperature variation to the saline conditions by assuming that b is approximately independent of density and replace $k_w(293.16 \text{ K})$ by $k_w(293.16 \text{ K}; \rho_w)$ as given by equation (A2). Similarly for viscosity we have for the fresh water temperature variation the formula

$$\mu_w(T) = \mu_r \left[\frac{\mu_w(293.16 \text{ K})}{\mu_r}\right]^{(293.16/T)^{3.102}} \qquad (A5)$$

where the reference viscosity $\mu_r = 10^{-4} \text{ kg m}^{-1}\text{ s}^{-1}$. We extend this expression to saline conditions by substituting $\mu_w(293.16 \text{ K}; \rho_w)$ as given by equation (A3) for the fresh water value at 293.16 K giving

$$\mu_w(T; \rho_w) = 10^{-4} \left[\frac{\mu_w(293.16 \text{ K}; \rho_w)}{10^{-4}}\right]^{(293.16/T)^{3.102}}, \qquad (A6)$$

ANEMOMETRIE A FILM CHAUD DANS LES SOLUTIONS AQUEUSES DE CHLORURE DE SODIUM

Résumé—On étudie expérimentalement l'étalonnage des sondes cylindriques à film chaud dans des solutions de NaCl. Les résultats sont compatibles avec la loi de Kramer si l'exposant du nombre de Reynolds est réduit à 0,36 et si une correction est faite pour la conduction de la fibre de quartz. Une loi modifiée de King semble fournir une approximation utile de la loi classique de Kramer pour les applications pratiques des jauges à film chaud aux écoulements à densité variable. Cette loi de King modifiée conduit à une méthode simple pour estimer les erreurs introduites quand les effets de la variation de densité sont ignorés dans l'étalonnage en vitesse d'une sonde à film chaud pour des solutions salines.

HEISSFILMANEMOMETRIE IN WÄSSRIGEN NaCl-LÖSUNGEN

Zusammenfassung—Zur Eichung von zylindrischen Heißfilmfühlern in wäßrigen NaCl-Lösungen wurde eine experimentelle Untersuchung durchgeführt. Die Resultate sind in Übereinstimmung mit dem Gesetz von Kramer, vorausgesetzt der Exponent der Reynolds-Zahl wird auf 0,36 vermindert und eine Korrektur zur Berücksichtigung der Wärmeleitung an die Quarzfaserschicht wird vorgenommen. Ein modifiziertes Kingsches Gesetz scheint eine brauchbare Näherung für das gebräuchliche Gesetz von Kramer bei der praktischen Anwendung von Heißfilmsonden in Strömungen variabler Dichte darzustellen. Das modifizierte Kingsche Gesetz liefert eine einfache Methode zur Abschätzung der Größenordnung des Fehlers, den man macht, wenn man den Einfluß variabler Dichte bei der Geschwindigkeitseichung von Heißfilmfühlern in Salzlösungen außer Betracht läßt.

ИСПОЛЬЗОВАНИЕ ПЛЕНОЧНОГО ТЕРМОАНЕМОМЕТРА ДЛЯ ПРОВЕДЕНИЯ ИЗМЕРЕНИЙ В ВОДНЫХ РАСТВОРАХ NaCl

Аннотация — Проведено исследование по калибровке цилиндрических пленочных датчиков в водных растворах NaCl. Полученные результаты согласуются с законом Крамерса при условии, что показатель степени при числе Рейнольдса не превышает 0,36. Сделана поправка на передачу тепла теплопроводностью к подложке остекленной пленки. Оказывается, что модифицированный закон Кинга является приемлемой аппроксимацией закона Крамерса для практических применений пленочных датчиков в потоках переменной плотности. Модифицированный закон Кинга дает простой метод оценки величины ошибок, обычно не учитываемых при калибровке пленочного датчика, когда влияние переменности плотности игнорируется.